CIRCULATORY FLOW IN A RECTANGULAR CAVITY AT MEDIUM AND HIGH REYNOLDS NUMBERS

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The study of circulatory flow induced in a rectangular cavity by the uniform translation of one of its boundaries requires accurate modeling of two-dimensional viscous flows (see, e.g., [1-4]). The main problem is that at large Reynolds numbers (here Reynolds number Re = U^*H^*/ν^*, where U is the velocity of the moving boundary, H is the length of the moving boundary, and ν is the coefficient of kinematic viscosity; the asterisk indicates dimensional quantities), starting from Re of the order of 10^3, the Navier–Stokes equations governing such flows contain diffusion term with a small coefficient. The application of finite difference methods to solve the Navier–Stokes equations introduces an artificial viscosity in the solution procedure due to errors of approximation which for certain schemes could exceed the real viscosity at relatively low Reynolds numbers (Re ~ 500), and these errors distort the solution to the problem. As established in [2], the first-order upwind schemes result in fluid flow with a certain effective Reynolds number which is appreciably no less than the true value, and this leads to a reduction in circulation with increase in Reynolds number [2]. The use of coarse grids, especially with uniformly distributed nodes in the computational region [5], leads to insufficient accuracy, and in a number of cases even erroneous results. Hence, there has been a tendency in recent times [4] to use schemes with higher-order approximations in combination with nonuniform grids whose nodes are clustered in regions of large gradients of critical flow parameters.

Plane rectangular cavities with different aspect ratios A (ratio of depth L* to width H*) are considered in the present paper. The unsteady Navier–Stokes equations are written in terms of vorticity and the stream function approach. A coordinate transformation is used to cluster grid lines near the wall region where the viscous parameters vary rapidly. The physical coordinates (x, y) are related to the transformed coordinates (ξ,η) by the following transformation:

$$z = D\ \{\zeta - b/(2\pi)\sin(2\pi\zeta)\},\ 0 \leqslant \zeta < 0.5,$$
$$z = D\ \{\zeta + b/(2\pi)\sin[2\pi(\zeta - 0.5)]\},\ 0.5 \leqslant \zeta \leqslant 1.$$

Here, for $z \equiv x$ and $\zeta \equiv \xi$, we have D = 1, and for $z \equiv y$, and $\zeta \equiv \eta$, we have D = A. All physical variables and length scales in the problem are nondimensionalized with respect to the characteristic quantities U* and H*.

The no-slip condition and the condition that the stream function is zero at solid boundaries are used as boundary conditions. The steady-state solution to the problem is obtained by solving the problem in time, the solution being started with the fluid in the cavity at rest.

The numerical solution of the vorticity transport equation is carried out through a step-by-step integration in time, using an explicit, second-order Adams–Bashforth triple deck scheme. The convective terms in this equation are represented by Arakawa schemes [6] of second and fourth orders, and the remaining spatial derivatives in the system of equations are approximated by central differences. The Gauss–Seidel iterative scheme with superior relaxation procedures is used for solving the equation that couples vorticity and stream function at each time step. Boundary conditions for vorticity are determined by second-order Woods' [2] scheme. The step size in time is established by numerical experiment.

It is worth mentioning that the approach used above has certain positive features as applied to typical flow characteristics. For a square cavity, it is well known [2] that with increasing Reynolds number the effect of viscosity on the flow is localized to the neighborhood of the cavity walls, whereas an inviscid core is formed in the central region. The use of finite-difference grids with small spatial interval near the walls makes it possible to

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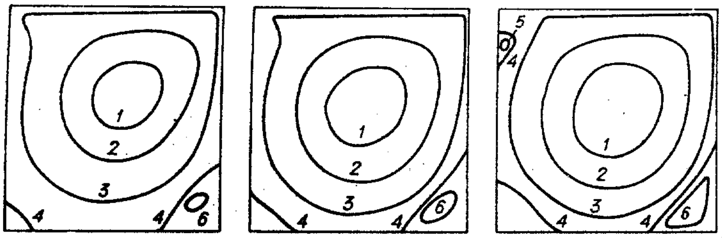

Fig. 1

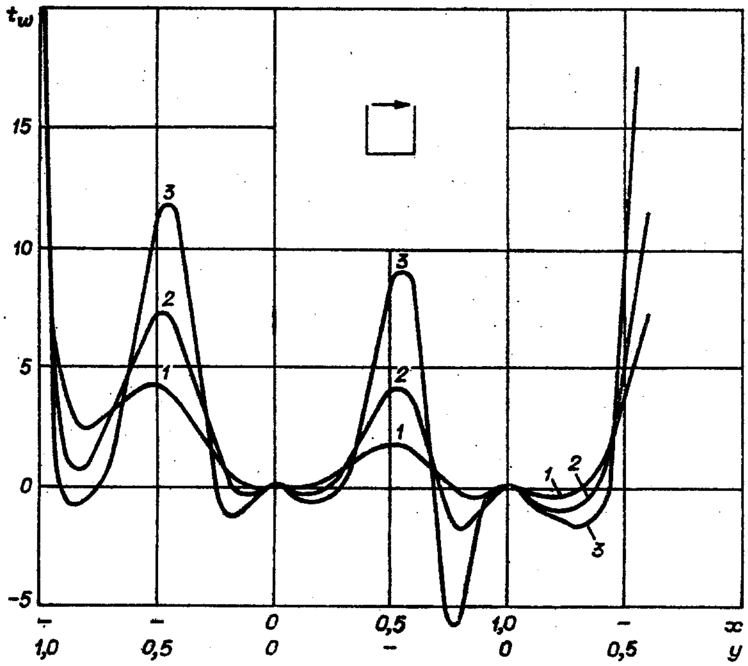

Fig. 2

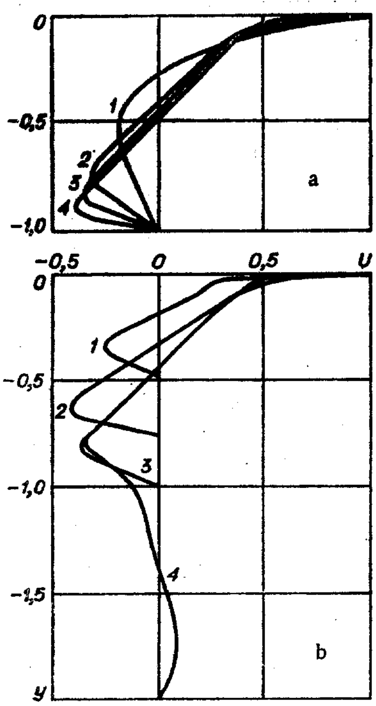

Fig. 3

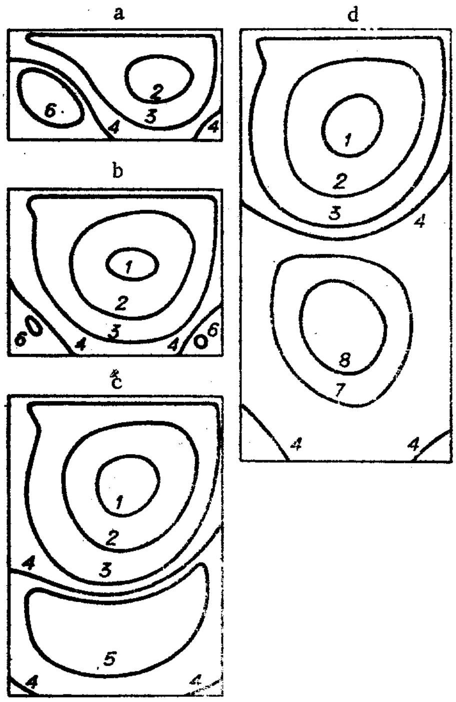

Fig. 4

accurately compute the viscous flow properties, and the application of the Arakawa scheme, which has the property of conserving mean values of the vorticity, the square of circulation, and the kinetic energy [6], conveys the structure of ideal flow without distortion (kinetic energy and the square of circulation are transported in the two-dimensional region from one grid point to the other without any artificial change).

Numerical computations of the viscous flows have been carried out for the square cavity for the range of Reynolds numbers from 100 to 2500, and also when Re = 1000 for square cavities with aspect ratios varying from 0.5 to 2.0. Stationary solutions obtained during the evolution of the flow with time have been analyzed. The number of time steps to reach the steady-state appears to be dependent on the dimensions of the region, Reynolds number, and the number of grid points, and this varies approximately from 1000 to 2500.

Streamlines in the square cavity are shown in Fig. 1a–c for Re = 400, 1000, and 2500, respectively (streamlines correspond to constant values of the stream function: curve 1) $\psi = -0.10$; 2) $\psi = -0.06$; 3) $\psi = -0.01$; 4) $\psi = 0$; 5) $\psi = 0.0001$; 6) $\psi = 0.001$). The flow structure for both medium and high Reynolds numbers is characterized by the presence of a developed central vortex and a pair of secondary corner vortices of lower strength. It is worth emphasizing that an increase in Reynolds number is accompanied by an increase in the strength of the secondary vortices at the lower corners of the cavity, and not a decrease as mentioned in [2]. The central vortex in the cavity grows but does not tend to occupy the entire cavity in the limiting case of the flow as $Re \to \infty$ (as assumed in [1]). It is possible to observe the tendency of the size of the corner vortices to stabilize at Reynolds numbers greater than 1000. An interesting feature of the flow structure is the onset and growth of the secondary vortex in the neighborhood of the moving boundary, starting from Re = 1500. This last result agrees well with the data on the flow structure given in [3, 4] for Reynolds numbers 2000 and 4000.

Figure 2 shows the distribution of skin friction on the walls of the square cavity for Re = 400; 1000; 2500 (curves 1-3, respectively). With increase in Reynolds number, an increase in the absolute value of skin friction is observed on all surfaces of the cavity, and the maximum friction at the bottom of the cavity increases almost proportional to the variation in Reynolds number. We observe the existence of appreciable friction in the zone of action of the secondary corner vortex near the downstream wall, and also a change in the sign of friction at the upstream wall close to the moving boundary.

Figure 3a shows profiles of the longitudinal velocity component in the central vertical plane of the square cavity for Re = 100, 400, 1000, and 25000 (curves 1-4, respectively). With increase in Reynolds number there is an increase in the velocity of the circulatory flow from 0.2 when Re = 100 to 0.41 when Re = 2500, and the maximum velocity approaches the bottom of the cavity. Thus, there is an intensification of the flow in the central vortex and an increase in its size. The analysis of velocity profiles confirms, in principle, Batchelor's idea [1] on the growth of the ideal core of constant vorticity in the central zone of the cavity and also on the localization of viscous effects in the small neighborhood of the cavity walls.

Figure 3b shows profiles of the longitudinal component of the velocity in the central vertical plane for Re = 1000 for rectangular cavities with A = 0.5, 0.75, 1.0, and 2.0 (curves 1-4, respectively). With increase in A from 0.5 to 0.75, the maximum value of the longitudinal velocity component increases and, when A is greater than 0.75, it decreases slightly. It is observed that for cavities with A equal to 1.0 and 2.0, the longitudinal velocity components in the central vortex are identical. The intensity of the flow in the secondary vortex in the cavity with A = 2.0 is much less than that in the primary vortex, the maximum value of the longitudinal velocity component not exceeding 0.06.

The streamline pattern is shown in Fig. 4a-d for Re = 1000 for rectangular cavities with A = 0.5, 0.75, 1.4, and 2.0 respectively (streamlines correspond to constant values of the stream function: curve 1) $\psi = -0.10$; 2) $\psi = -0.06$; 3) $\psi = 0.01$; 4) $\psi = 0$; 5) $\psi = 0.0001$; 6) ψ 0.001; 7) $\psi = 0.01$; 8) $\psi = 0.02$). For small depths of the cavity (A = 0.5), the flow structure is characterized by the presence of two large scale vortices: a well-developed central vortex occupying the larger portion of the region and a weaker vortex in the corner behind the upstream wall. The structure of the latter vortex is, similar to a certain extent, to the flow structure in the wake of a bluff body. The secondary vortex near the rear wall is a small-scale vortex and it is weak and small. With an increase in the depth of the cavity from 0.5

to 1.0 (Fig. 4a, b and Fig. 1b) there is an increase in the size and strength of the central vortex and also a gradual rearrangement of the structure of the corner vortices, exhibiting a reduction in the vortex size at the upstream wall and an increase at the downstream wall. With further increase in the cavity depth (Fig. 4c, d), the structural evolution of the central vortex ends, and the vortex size and strength are stabilized and no longer depend on the change in depth beyond A = 1.0. However, corner vortices continue to grow as before and then merge into one another, forming a single secondary vortex which occupies the entire width of the cavity and rolls to the side opposite to the direction of rotation of the central vortex. An increase in A is accompanied by a strengthening of the secondary vortex, and the flow velocity in it is increased but still remains much weaker than the central vortex. The formation of the vortices in the lower corners of the cavity and their growth with change in cavity depth have been observed for cavities with A = 1.4 and 2.0. The latter case agrees with the experimentally observed tendency in a deep cavity for the growth of a system of vertically located large-scale vortices whose intensity subsides as the bottom of the cavity is approached. Vortices are separated from each other by a fairly wide merging zone, while, as indicated by analysis of the results presented in Fig. 3b and 4, it is also possible to identify the core of constant vorticity in the secondary vortex.

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